Fuzzy random reliability analysis of aseismic structures

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ABSTRACT, Since earthquake loads and structural resistances possess both fuzziness and randomness, the reliability analysis for aseismic structure is then a fuzzy-random problem. In this paper, a method of fuzzy-random reliability analysis for aseismic structures with multiple failure modes is put forward, and an initial research on the reliability based optimum design for them is done. To do this, some concepts and definitions such as fuzzy earthquake intensity, fuzzy response of structure, satisfaction degree to the fuzzy safe criterion, fuzzy safe and unsafe regions are proposed.

1 FUZZY-RANDOM FACTORS IN RELIABILITY ANALYSIS OF ASEISMIC STRUCTURES

The probability for a structure to work normally under design condition within the stipulated working period T is called the reliability of the structure. By following analysis, it can be known that the earthquake loads and structural resistances possess obvious fuzziness and randomness. So, what is called " to work normally " in the above definition is in fact a fuzzy-random event.

When a structure is given, its response caused by earthquake loads depends on the earthquake intensity and the site soil classification of the building site. The assessment of the maximum earthquake intensity at the building site during the service life T of a structure is the task of earthquake risk analysis which is beyond the scope of this paper. We only use its method in the reliability analysis to get the probabilities $P(I_i)$ of the maximum earthquake intensity I_i (i=6, 7, 8, 9, 10) ocurring at the building site during the service life T. In the engineering point of view, we need only to consider the case of

$$\sum_{i=6}^{10} P(I_i) = 1 \qquad (1)$$

because for structures in zones with predictive intensity lower than I_6, the effect of earthquake need not to be considered, and it is impermissible to build important structures in zones with pridictive intensity higher than I_{10}. In this way, the randomness of earthquake intensity is considered.

As a comprehensive measure of the severity of earthquake, the intensity must changes gradually and have a continuous universe of discourse. But in order to make full use of the research results in earthquake risk analysis, the intensity scale with 12 degree may be still used, but each intensity degree I_i is regarded as a fuzzy subset $\underset{\sim}{I}_i$ on the continuous intensity universe of discourse [0,12]. We suggest (see Wang and Wang (1985)) that the membership function of the fuzzy intensity degree $\underset{\sim}{I}_i$ have the form

$$\mu_{I_i}(I) = [\sin(I-I_i+0.5)\pi+1] \qquad I \in [I_i-1,\ I_i+1] \qquad (2)$$

where I_i is the ordinal number for $\underset{\sim}{I}_i$ (I_i is equal to i in value) and $\mu_{I_i}(I)$ is schematically illustrated in Fig.1.

The seismic acceleration response spectrum A(T) stipulated by current Chinese aseismic design code is shown in Fig.2 (in gravity acceleration g), where the parameters A_m and T_0 depend on the intensity and the site soil grade respectively as shown in table 1. The horizontal coordinate T is the natural period of the vibration mode of the structure.

As descrete intensity universe of dis-

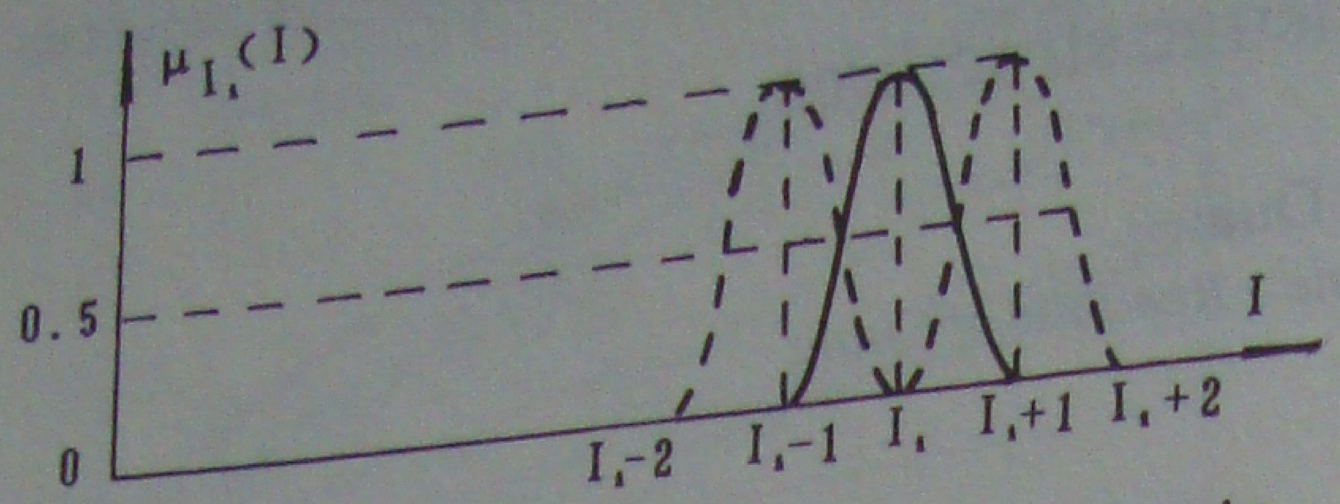

Figure 1. Membership function of fuzzy intensity degree I_i

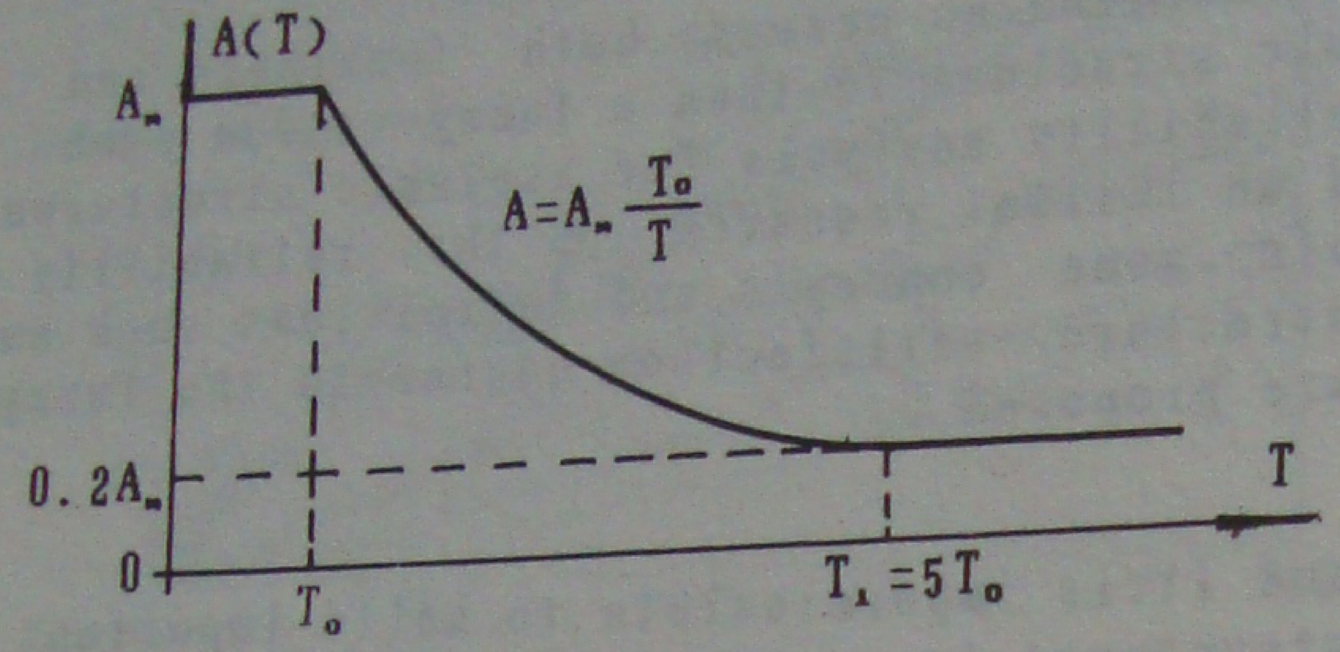

Figure 2. Seismic acceleration response spectrum A(T) in Chinese code

Table 1.

Intensity degree I_i	7	8	9
A_m (g)	0.23	0.45	0.90
Site soil grade	I	II	III
T_g (sec.)	0.2	0.3	0.7

course is altered into continuous one, the relation between intensity I and coefficient A_m in table 1 becomes

$$A_m(I) = 0.9\times 2^{I-9} \tag{3}$$

or

$$I(A_m) = 1.4427\ln A_m + 9.152 \tag{4}$$

According to this relation, the membership function of fuzzy parameter $\underset{\sim}{A_m}$ corresponding to intensity $\underset{\sim}{I_i}$ can be derived as

$$\mu_{A_m}(A_m) = [\sin(1.4427\ln A_m + 9.652 - I_i)\pi + 1]$$
$$A_m \in [0.9\times 2^{I_i-10},\ 0.9\times 2^{I_i-8}] \tag{5}$$

Since the building site has been chosen before designing, site soil classification has only fuzziness without randomness. In this case, in order to simplify the problem, Wang and Wang (1985) suggested that the procedure of fuzzy comprehensive evaluation should be adopted to judge the site soil grede vector.

$$\underset{\sim}{B} = b_1/\text{I} + b_2/\text{II} + b_3/\text{III} \tag{6}$$

where b_1, b_2 and b_3 represent the membership degrees of the site to the site soil grade I, II and III respectively. Now the comprehensively judged value of the fuzzy parameter $\underset{\sim}{T_g}$ can be obtained by the weighted mean method

$$T_g = \frac{\sum_{k=1}^{3} b^2_k T_{gk}}{\sum_{k=1}^{3} b^2_k} \tag{7}$$

in which T_{gk} is the value of T_g when the site soil grade is k, i.e., $T_{g1}=0.2$, $T_{g2}=0.3$ and $T_{g3}=0.7$ sec. according to table 1.

Having the "fuzzy response spectrum" with fuzzy parameters $\underset{\sim}{A_m}$ and $\underset{\sim}{T_g}$ ($\underset{\sim}{T_g}$ is replaced approximately by the comprehensively judged value T_g), the fuzzy earthquake loads can be obtained according to the Code.

Structural resistances are related to the safe criterion for structure. Since it is reasonable that there should be a transition stage from absolute allowable to absolute unallowable states for any response "S" of the structure, the corresponding allowable interval "$\underset{\sim}{R}$" for the response "S" should be fuzzy. The typical form of its membership function is shown in Fig. 3. To simplify the problem, the difference in quantity resulted from the randomness of the structural resistance can be combined with the influence of its fuzziness. So, we can approximately consider that the allowable interval $\underset{\sim}{R}$ for the response S has only fuzziness.

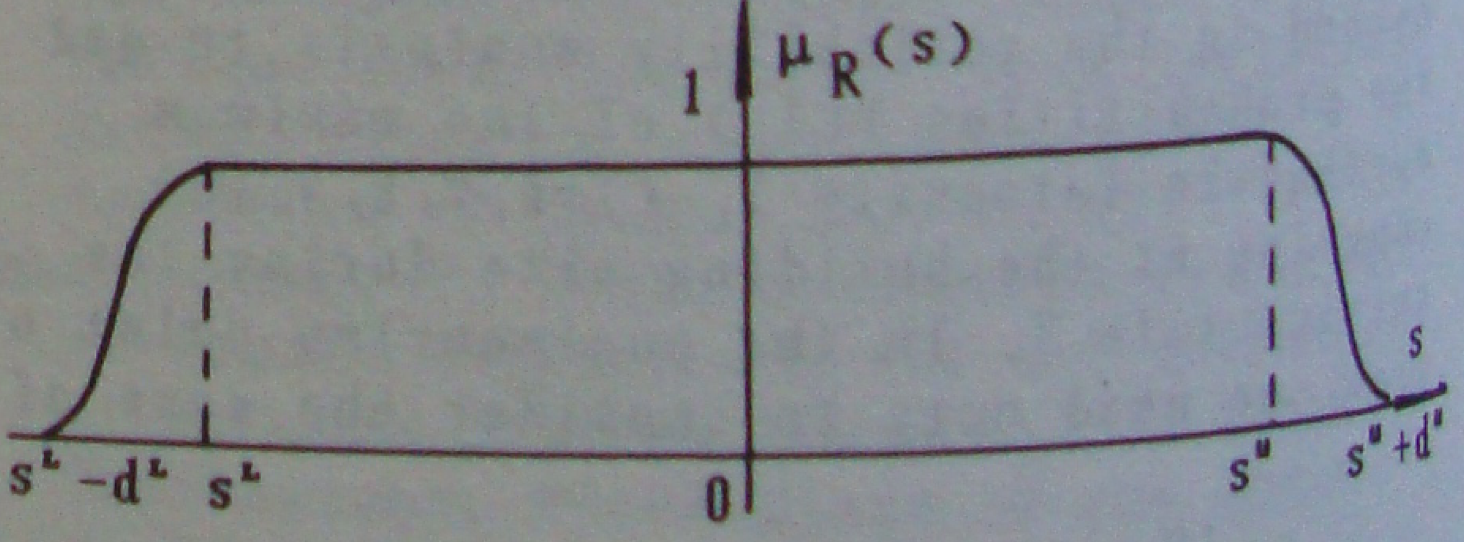

Figure 3. Membership function of fuzzy allowable interval $\underset{\sim}{R}$

2 SATISFACTION DEGREE TO FUZZY CONSTRAINTS

In structural reliability analysis, the

design scheme $\bar{x}$ of the structure is known. In general, the maxmum value of some behavior responses (e.g., stresses, displacements, etc.) of the structure are used as the limits of the criterion to judge if the structure works normally. The maximum values of those responses r_j are represented by S_j $(j=1,\ldots,J)$. For structure with multiple failure modes, $J>1$.

According to the current code, the maximum response S_j of the structure subjected to non-fuzzy seismic load (definite response spectrum) is proportional to A_m

$$S_j = K_j A_m \qquad (8)$$

where K_j is the maximum value of response r_j when $A_m=1$, which is a constant and may be evaluated by using the items of the Code.

According to the principle of extension in fuzzy mathematics, and Eq.(8), when $\underset{\sim}{A}_m$ is a fuzzy number, the membership function of fuzzy maximum response $\underset{\sim}{S}_{ji}$ under fuzzy intensity degree $\underset{\sim}{I}_i$ should be

$$\mu_{S_{ji}}(s_j) = \left[\sin\left(1.4427\ln\frac{S_j}{K_j}+9.652-I_i\right)+1\right]$$

$$s_j \in [0.9K_j\times 2^{I_i-8},\ 0.9K_j\times 2^{I_i-10}] \qquad (9)$$

It is schematically shown in Fig. 4.

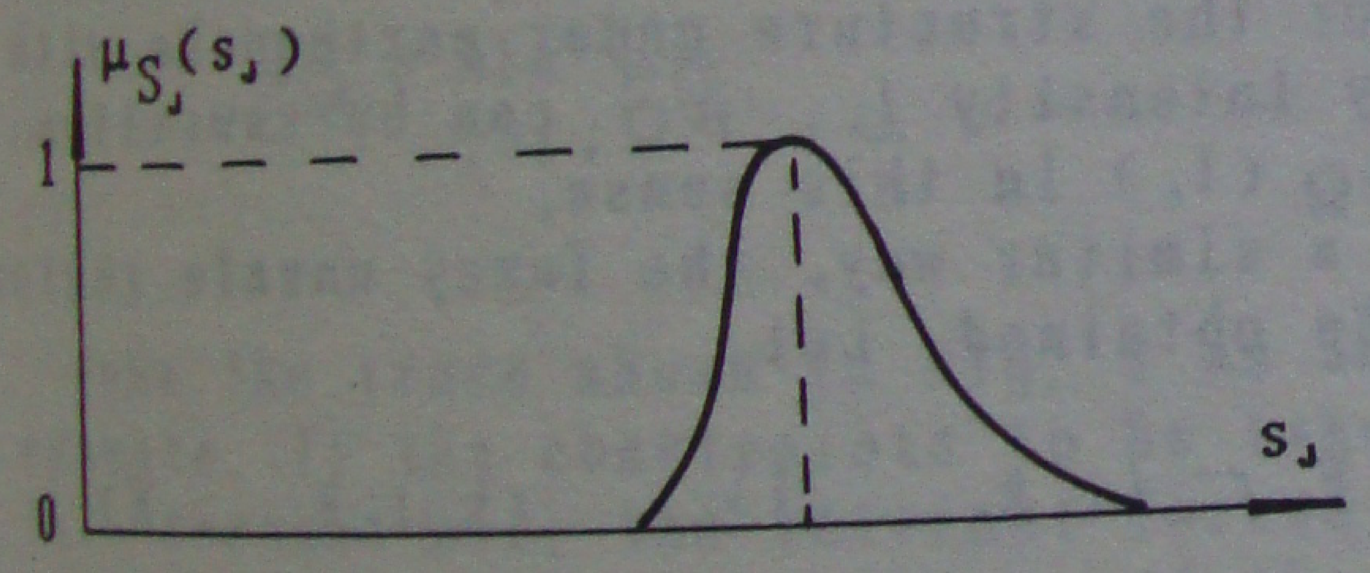

Figure 4. Membership function of fuzzy maximum response $\underset{\sim}{S}_j$

The fuzzy event that structure works normally under earthquake with fuzzy intensity $\underset{\sim}{I}_i$ is actually a fact to satisfy a group of fuzzy constraints

$$\underset{\sim}{\Omega}_{ji} \triangleq \{\underset{\sim}{S}_{ji} \subset \underset{\sim}{R}_j\} \qquad (j=1,2,\ldots,J) \qquad (10)$$

where $\underset{\sim}{R}_j$ is the fuzzy allowable interval of the maximum response $\underset{\sim}{S}_j$.

Since the group of constraints Eq.(10) is fuzzy, the level of satisfaction to it may be different. This level may be called "satisfaction degree" and denoted by β_{ji} $(j=1,2,\ldots J)$. Therefore, the fuzzy constraint $\underset{\sim}{\Omega}_{ji}$ stands for a fuzzy event that the fuzzy maximum response $\underset{\sim}{S}_{ji}$ falls into the fuzzy allowable interval $\underset{\sim}{R}_j$ in the sense of having different satisfaction degree β_{ji} $(0<\beta_{ji}\leq 1)$.

The satisfaction degree β_{ji} to the fuzzy constraint $\underset{\sim}{\Omega}_{ji}$ is also the membership degree $\mu_{\Omega_{ji}}$ of the fuzzy maximum response $\underset{\sim}{S}_{ji}$ to the fuzzy event $\underset{\sim}{\Omega}_{ji}$ when the structure is exposed to the earthquake with fuzzy intensity $\underset{\sim}{I}_i$. Obviously, β_{ji} relates directly to the intensity $\underset{\sim}{I}_i$ to be considered. The value of satisfaction degree β_{ji} depends on the relative position of the membership function curves μ_S and μ_R of fuzzy maximum response $\underset{\sim}{S}_{ji}$ and its fuzzy allowable interval $\underset{\sim}{R}_j$ respectively (see Fig. 5). When μ_S is covered entirly by the interval of μ_R =1 (figure a) the constraint $\underset{\sim}{\Omega}_{ji}$ is satisfied completely, $\mu_{\Omega_{ji}}=1$; when μ_S is located out of μ_R entirly (Fig.c), the constraint $\underset{\sim}{\Omega}_{ji}$ is not satisfied absolutely, $\mu_{\Omega_{ji}}=0$; while μ_S and μ_R overlap each other (Fig.b), the constraint $\underset{\sim}{\Omega}_{ji}$ is satisfied to a certain extent, $\mu_{\Omega_{ji}} \in [0,1]$. Therefore, we suggest to define

$$\mu_{\Omega_{ji}} = \frac{\int_{-\infty}^{+\infty}\mu_R(s_j)\,\mu_S(s_j)\,ds_j}{\int_{-\infty}^{+\infty}\mu_S(s_j)\,ds_j} \qquad (11)$$

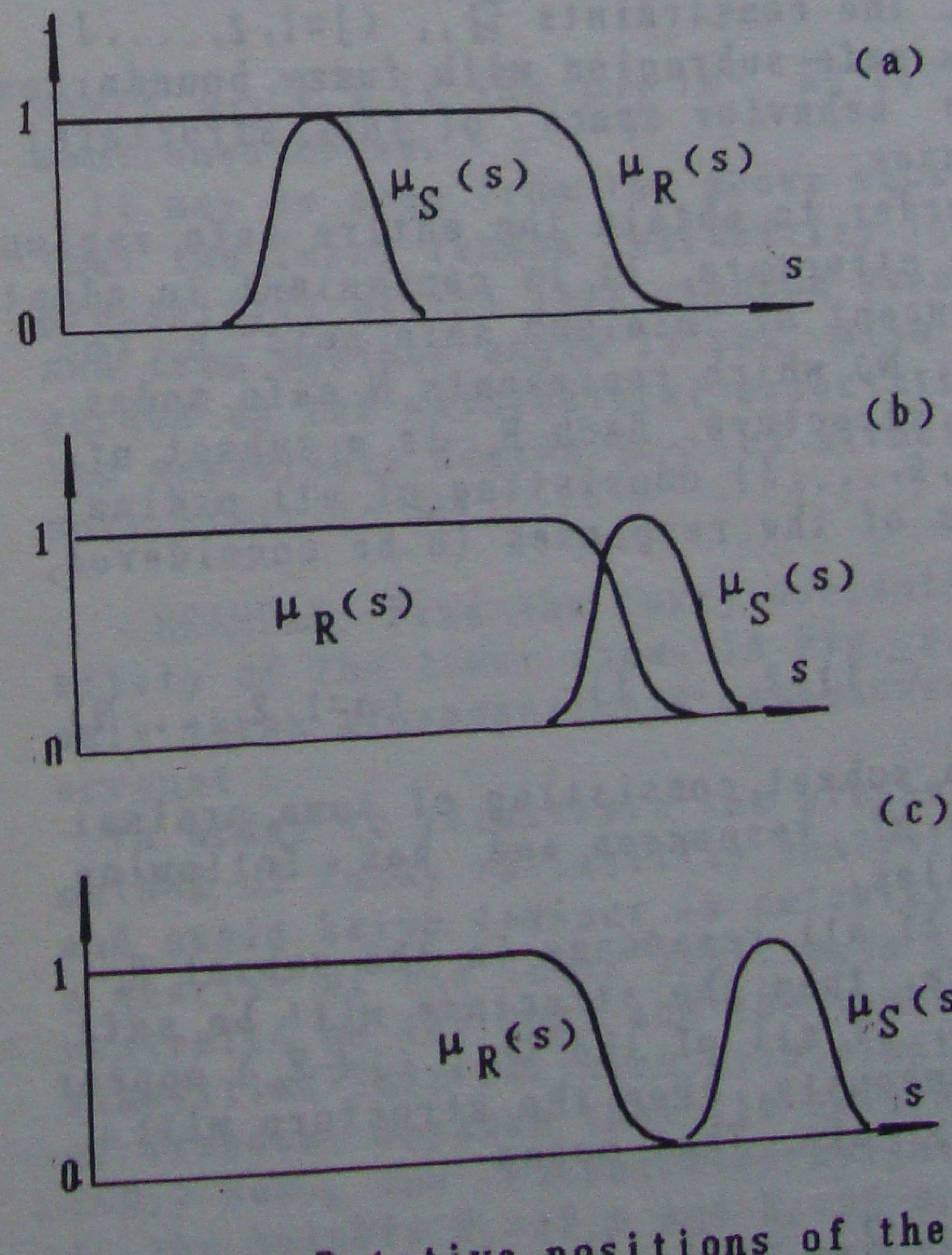

Figure 5. Relative positions of the membership function curves of fuzzy maximum response and its fuzzy allowable interval

When the curves of the transition stages of $\mu_R(s_j)$ adopt inclined straight lines, the following integral will be found in calculation of the numerator of Eq.(11):

$$\int a^I \times [\sin(I-I_i+0.5)\pi+1]\,dI =$$

$$\frac{a^I}{\ln a}\left[1+\frac{\sin(I-I_i+0.5)\pi-(\pi/\ln a)\cos(I-I_i+0.5)\pi}{1+\left(\frac{\pi}{\ln a}\right)^2}\right] \quad (12)$$

where "a" is a constant. The denorminator in Eq.(11) is

$$\int_{-\infty}^{+\infty}\mu_S(s_j)ds_j = 0.6437\times 2^{I_i-9}K_j \quad (13)$$

The calculation will be simplified by using these formulas.

3 FUZZY SAFE AND UNSAFE REGIONS OF STRUCTURE

When the structure has multiple failure modes, the fuzzy event that structure works normally is in fact to require that some or all of the constraints

$$\underset{\sim}{\Omega}_{ji} \triangleq \{\underset{\sim}{S}_{ji} \subseteq \underset{\sim}{R}_j\}\ (j=1,2,\ldots,J) \quad (14)$$

are satisfied to different extents, i.e., to require that some or all of the fuzzy maximum responses $\underset{\sim}{S}_{ji}$ fall into their fuzzy allowable intervals $\underset{\sim}{R}_j$ respectively with different satisfaction degrees. In this sense, the constraints $\underset{\sim}{\Omega}_{ji}$ $(j=1,2,\ldots,J)$ form a safe subregion with fuzzy boundaries in the behavior space of the structural responses.

In order to obtain the entire safe region of the structure, it is convenient to adopt the concept of "minimum safe sets" E_n $(n=1,2,\ldots,N)$ which represents N safe modes of the structure. Each E_n is a subset of set $\{1,2,\ldots,J\}$ consisting of all ordinal numbers of the responses to be considered, i.e.,

$$E_n \subset \{1,2,\ldots,J\}, \quad (n=1,2,\ldots,N)$$

It is a subset consisting of some ordinal numbers of responses and has following properties:

(1). If all responses in the subset E_n are safe, then the structure will be safe. That is, if all of the $\underset{\sim}{\Omega}_{ji}$ $(j\in E_n)$ appear simultaneously, then the structure will be fuzzy safe. That means

$$\underset{\sim}{\Omega}_{in} = \bigcap_{j\in E_n}\underset{\sim}{\Omega}_{ji} \quad (15)$$

In this case, no matter whether other responses $(j\bar{\in} E_n)$ are safe or unsafe, it makes no difference to the safety of the structure. Therefore, E_n represents the nth safe mode of the structure and $\underset{\sim}{\Omega}_{in}$ is the nth fuzzy safe subregion under earthquake with fuzzy intensity $\underset{\sim}{I}_i$.

(2). If any component (ordial number of the responses) is removed from the subset E_n, then the property (1) will cease to be valid, so the subset is refered to as "minimum set".

Since any safe state must at least include a certain minimum safe set, and any safe subregion implies that the structure is in safe state, so the entire fuzzy safe region of the structure is the union of N fuzzy safe subregions, i.e.

$$\underset{\sim}{\Omega}_i = \bigcup_{n=1}^{N}\underset{\sim}{\Omega}_{in} = \bigcup_{n=1}^{N}\left(\bigcap_{j\in E_n}\underset{\sim}{\Omega}_{ji}\right) \quad (16)$$

This is just the entire fuzzy safe region of the structure under earthquake with fuzzy intensity $\underset{\sim}{I}_i$.

According to the basic operation rules of fuzzy sets, the membership degree to the fuzzy safe region $\underset{\sim}{\Omega}_i$ for a structure can be obtained from Eq.(16):

$$\mu_{\Omega_i} = \max_{n=1}^{N}\left[\min_{j\in E_n}\mu_{\Omega_{ji}}\right] \quad (17)$$

which is the membership degree of the work state (or responses) to normal work state $\underset{\sim}{\Omega}$ for the structure under earthquake with fuzzy intensity $\underset{\sim}{I}_i$. μ_{Ω_i} can be rewritten as $\mu_\Omega(I_i)$ in this sense.

In a similar way, the fuzzy unsafe region can be obtained. Let

$$F_k \subset \{1,2,\ldots,J\} \quad (k=1,2,\ldots,K)$$

stand for kth "minimum unsafe set", it is also a subset composed of some ordinal numbers of responses. If all of the responses in this subset are in failure states simultaneously, the structure will fail. F_k stands for kth failure mode which corresponds to a fuzzy unsafe subregion. The union of all these fuzzy unsafe subregions constitutes the entire fuzzy unsafe region of the structure. Thus, following equation can be derived:

$$\mu_{\Omega_i} = 1-\max_{k=1}\left[\min_{j\in F_k}(1-\mu_{\Omega_{ji}})\right] \quad (18)$$

Eq.(17) and (18) are equivalent.

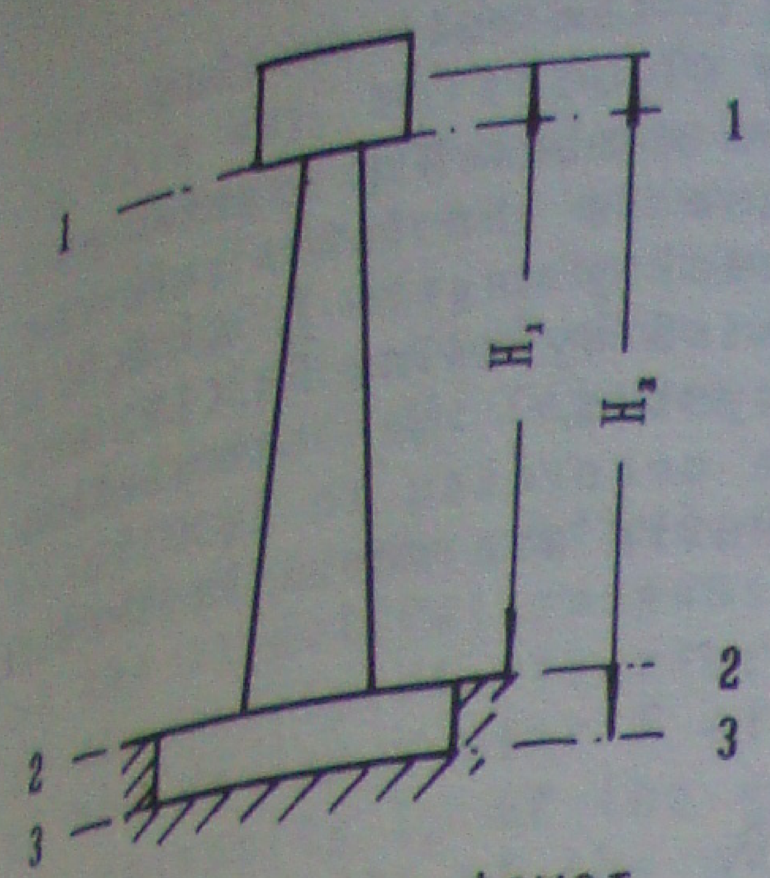

Figure 6. Water tower

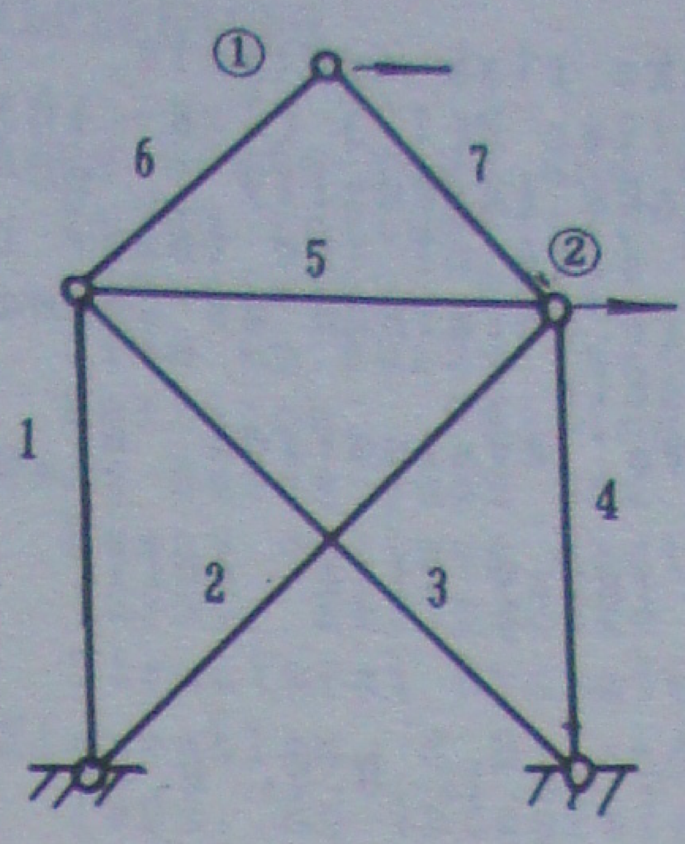

Figure 7. Seven- bar truss

Take a water tower shown in Fig.6 as an example. If in the reliability analysis we only take account of its following three responses:

(1). the shear force in cross-section 1-1 ($r_1=Q_1$)

(2). the bending moment in cross-section 2-2 ($r_2=M_2$)

(3). the overturning moment about the bottom of the base 3-3 ($r_3=M_3$).

Then it is a simple series system. Its ordinal number set of response is {1, 2, 3}. This system has 3 minimum fuzzy unsafe sets (K=3), $F_1=\{1\}$, $F_2=\{2\}$, $F_3=\{3\}$; and only one minimum safe set (N=1), $E_1=\{1, 2, 3\}$.

In general, for any series system, there is only one minimum safe set E={1, 2, ...J}. In this case, the formulas (17) and (18) are simplified into a same form:

$$\mu_{\Omega_1} = \mathop{Min}_{j=1}^{J} \mu_{\Omega_{j1}} \qquad (19)$$

Take the truss shown in Fig. 7 as another example. If the constraints to be taken into account are: the internal forces r_j (j =1,2,...,7) of the seven bars and the horizontal displacements r_j (j=8, 9) of joints ① and ② can not exceed their corresponding allowable values, then there would be 14 (K=14) minimum unsafe sets $F_1=\{6\}$, $F_2=\{7\}$, $F_3=\{8\}$, $F_4=\{9\}$, $F_5=\{1, 2, \}$, $F_6=\{1, 3\}$, $F_7=\{1, 4\}$, $F_8=\{1, 5\}$, $F_9=\{2, 3\}$, $F_{10}=\{2, 4\}$, $F_{11}=\{2, 5\}$, $F_{12}=\{3, 4\}$, $F_{13}=\{3, 5\}$, $F_{14}=\{4, 5\}$.

This example has 5 (N=5) fuzzy minimum safe sets, in which each set will include set {6, 7, 8, 9} and any other four out of the five ordinal numbers of 1～5.

As for complex structures, their minimum safe sets and minimum unsafe sets may be determined by using system analysis techniques, such as the fault-tree method.

4 FUZZY-RANDOM RELIABILITY OF ASEISMIC STRUCTURES

Assume that the probabilities $P(I_i)$ (i=6,... 10) of the earthquake with maximum intensity I_i occuring at the building site during the service life T of a structure are known from earthquake risk analysis, then after having obtained the membership degrees $\mu_{\underset{\sim}{\Omega}}(I_i)$ of work state to the normal work $\underset{\sim}{\Omega}$ for the structure under earthquake intensities I_i (i=6,...,10) respectively, the probability that the structure can work normally in its service life T can be found by the method of fuzzy probability theory. Thus, the fuzzy random reliability of the structure will be

$$\Psi = P(\underset{\sim}{\Omega}) = \sum_{i=6}^{10} \mu_{\underset{\sim}{\Omega}}(I_i)P(I_i) \qquad (20)$$

where $\mu_{\underset{\sim}{\Omega}}(I_i)$ is the μ_{Ω_i} in Eq.(17) as mentioned above.

It may be seen from the above equation that the fuzzy-random reliability is the probability of the fuzzy-random event $\underset{\sim}{\Omega}$, and from another angle it can also be regarded as the mathematical expectation of the membership degree $\mu_{\underset{\sim}{\Omega}}$ of structural work state to $\underset{\sim}{\Omega}$.

EXAMPLE. Find the fuzzy aseismic reliability of the tower shown in Fig. 6. Take only three aforementioned responses into account.

In order to illustrate the presented method of fuzzy reliability analysis and avoid being tedious in calculation, a quarter of the tower shaft mass is concentrated on the top, and the tower is thus simplified into a system with single degree of freedom. Suppose that its natural period T=0.7 sec., the concentrated weight W=180 t, the heights H_1=20 m and H_2=22 m.

The calculation is carried out as following:

(1). Making earthquake risk analysis.

Assume the probabilities $P(I_i)$ $(i=6\sim10)$ of the maximum earthquake intensity $\underset{\sim}{I}_i$ occuring at the building site of the tower in its service life T are obtained as shown in the second row of table 2.

(2). The comprehensive evaluation of the site soil grade.

Suppose the obtained fuzzy site soil grade vector is

$$\underset{\sim}{B} = [b_1,\ b_2,\ b_3\ ] = [0.1,\ 0.5,\ 0.8]$$

According to Eq.(7), the comprehensively evaluated value of the parameter T_g is

$$T_g = \frac{0.1^2\times0.2\ +0.5^2\times0.3+0.8^2\times0.7}{0.1^2+0.5^2+0.8^2}$$

$= 0.583$ sec.

(3). Calculating the maximum responses K_j of the tower.

Since T_g is less than the natural vibration period of the tower, the seismic response coefficient can be obtained from Fig.2 and table 1.

$$A(T) = A_m T_g/T = 0.833A_m$$

The structure coefficient is taken as C =0.5 according to the Code. In this way, the maximum values of shear force Q_1, bending moment M_2 and overturning moment M_3 can be obtained,

$$S_1 = CA(T)W = 0.5\times0.833\times180A_m = 74.96A_m$$
$$S_2 = CA(T)WH_1 = 1499.2A_m$$
$$S_3 = CA(T)WH_2 = 1649.1A_m$$

Then, the maximum responses when $A_m=1$ are,

$K_1 \approx 75$ t, $K_2 \approx 1500$ tm, $K_3 \approx 1650$

(4). Giving the membership functions $\mu_{R_j}(s_j)$.

According to the circumstances around the structure and the requirements for normal work of the structure, the membership functions $\mu_{R_j}(s_j)$ of the fuzzy allowable intervals $\underset{\sim}{R}_j$ corresponding to the maximum responses $\underset{\sim}{S}_j$ $(j=1,2,3)$ are given in Fig.8.

(5). Calculating the satisfaction degrees $\mu_{\Omega_{j,i}}$.

All satisfaction degree $\mu_{\Omega_{j,i}}$, i.e., $\mu_{\Omega_j}(I_i)$ for maximum responses $\underset{\sim}{S}_j$ $(j=1,2,3)$ corresponding to each intensity $\underset{\sim}{I}_i$ can be obtained by using Eqs (9) and (11), and Eqs (12) and (13) may be used to simplify the integral calculation. The obtained results are shown in the 3rd, 4th and 5th rows of table 2.

(6). Calculating the satisfaction degrees $\mu_\Omega(I_i)$.

In general, these membership degrees of the maximum response to the fuzzy safe region for each intensity degree $\underset{\sim}{I}_i$ $(i=6,7,\ldots,10)$ can be calculated by using Eq.(17) or (18). For series system, the calculation can be simply done according to Eq.(19). The calculated results are shown in the last row of table 2.

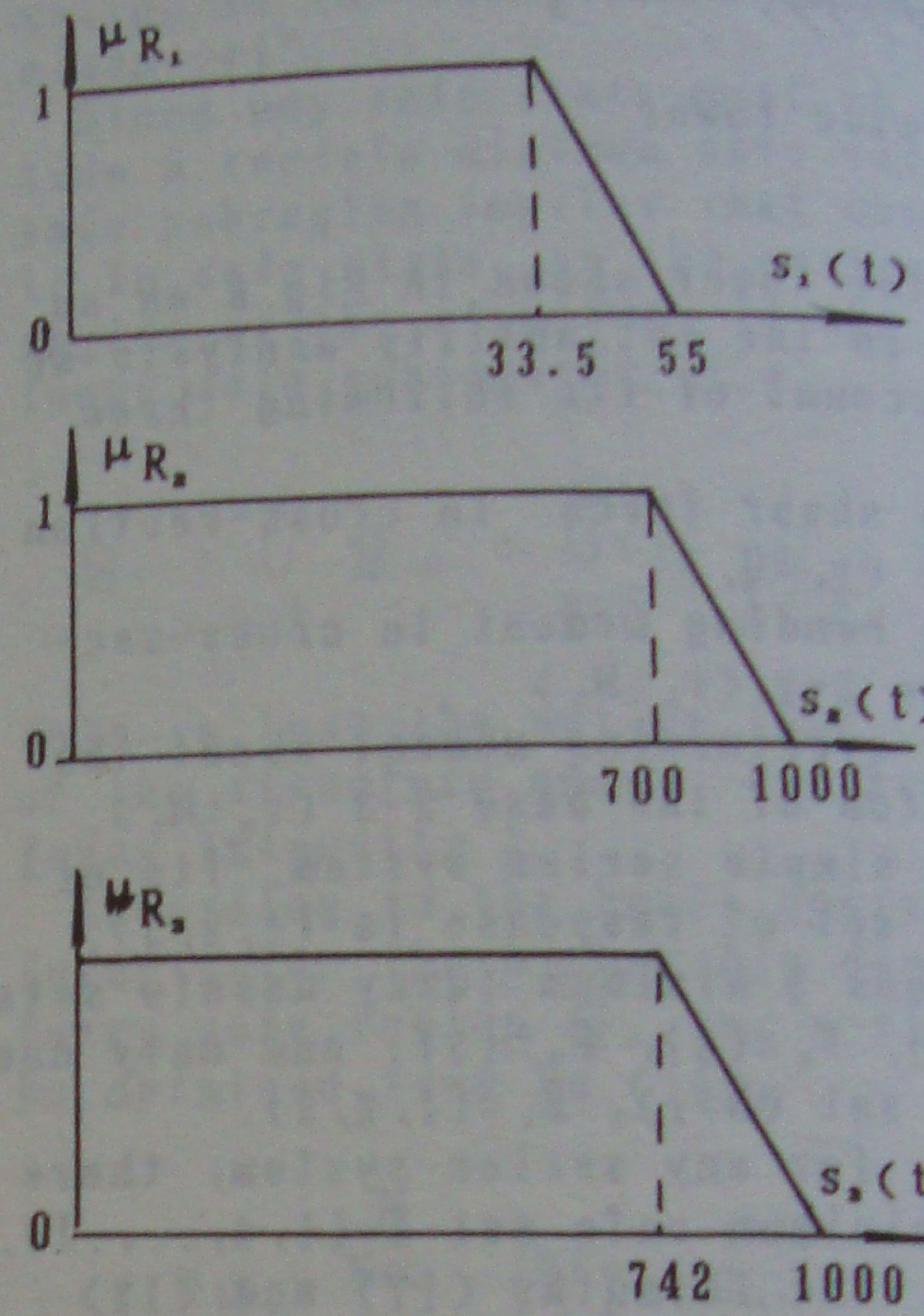

Figure 8. Membership functions of the fuzzy allowable intervals of the responses

Table 2.

I_i	6	7	8	9	10
$P(I_i)$	0.46	0.40	0.10	0.03	0.01
$\mu_{\Omega_{1,i}}$	1.000	0.999	0.741	0.044	0.000
$\mu_{\Omega_{2,i}}$	1.000	1.000	0.725	0.026	0.000
$\mu_{\Omega_{3,i}}$	1.000	1.000	0.620	0.022	0.000
$\mu_\Omega(I_i)$	1.000	0.999	0.620	0.022	0.000

(7). calculating the fuzzy reliability of the structure.

According to Eq.(20), it is

$$\Psi = 0.46\times1.00 + 0.40\times0.999 + 0.10\times0.620 + 0.03\times0.022 + 0.01\times0.000 = 0.922$$

5 BASIC CONCEPT OF RELIABILITY BASED FUZZY OPTIMUM DESIGN OF STRUCTURES

The more rational optimum design of structures should be based on the analysis of structural reliability, for the function of structure as a whole may be considered only in this way, while each strength-constraint in ordinary optimum design is considered locally from every element. Especially, in a structure or substructure of parallel system, failure of some elements will not always result in failure of the whole structure.

Obviously, the satisfaction degrees $\mu_{\Omega_{j,i}}$ of the structural responses under earthquake to the safety of structure are functions of the design vector $\bar{x}$ of the structure and the intensity I_i of the earthquake. So, the reliability of the structure is also a function of $\bar{x}$,

$$\Psi(\bar{x}) = \sum_{i=6}^{10} \mu_{\Omega}(I_i, \bar{x}) P(I_i) \qquad (21)$$

in which

$$\mu_{\Omega}(I_i, \bar{x}) = \max_{n=1}^{N} \left[\min_{j \in E_n} \mu_{\Omega_{j,i}}(I_i, \bar{x}) \right] \qquad (22)$$

Then, a mathematical model of a fuzzy optimum design based on reliability analysis is

Find $\bar{x}$, to

$$\text{minimize } W(\bar{x}) = C(\bar{x}) + E[\Psi(\bar{x})] \qquad (23)$$

Subjected to $\Psi(\bar{x}) \supseteq \Psi^{L}$

Where $C(\bar{x})$ is initial fabrication cost of the structure, $E[\Psi(\bar{x})]$ is the loss expectation when the structure is damaged during its service life, Ψ^{L} is a fuzzy lower bound to the structural reliability.

6 CONCLUSION

The fuzzy and random factors in the earthquake intensity, the site soil classification and the allowable intervals of structural responses have been taken into account in the reliability analysis of aseismic structures in this paper. The concept of the structural reliability may be more extended. The current definition and concept of reliability only deal with the randomness of things, but in fact, any uncertain factor which exists in the structures or in the external loads would lead to some uncertainty to the safety of structures and consequently lead to reliability problem. Thus, a cocept of generalized reliability and its calculation method are proposed in our another paper.

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